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ORDERED PEAK STATISTICS THROUGH DIGITAL SIMULATION

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SUMMARY

A detailed knowledge of the seismic response time history in the form of the first, second, third, ... largest peak amplitudes is useful to understand the progressive damage in a structure. This information is however not available from the conventionally used response spectra. This paper provides a formulation of the statistics of these higher order peaks by proposing a digitally simulated joint density function for the peaks in a stationary, Gaussian process. A digital experimentation has been done by generating an ensemble of the single-degree-of-freedom oscillator response to several artificial accelerograms, and the corresponding ordered peak distributions have been found to be in close agreement with the predictions based on the formulation in this paper. The peak factors based on the proposed formulation have also been compared with those from the existing formulations based on the Markov theory and on the assumption of peak independence. It has been found that whereas the peak factors from the proposed and Markovian formulations are compatible, the assumption of independence may give reasonable estimates only for the first few orders of peaks.

KEY WORDS: random processes; seismic response; peak factors; order statistics; peak dependence; simulated joint density

1. INTRODUCTION

In earthquake resistant design, the structures are made ductile to undergo the damaging response without failure, and the reductions are permitted in the linear elastic design forces, for economical reasons, due to the assumed large energy dissipation potential through the inelastic deformations. The design practices based on this philosophy involve the estimation of the largest peak amplitudes using the response spectra, and ignore the complete inelastic deformation time history, which includes the number, sequence and the relative amplitudes of the inelastic excursions. However, these repeated inelastic excursions may also have appreciable effects on the cumulative damage of the structures. Thus, to estimate the likelihood of failure of a given structure, particularly in response to the random phenomenon like earthquake excitation, it becomes necessary to study the statistics of the ordered peaks in the response process. The ordered peak statistics are also useful in visualizing the role, which higher order peaks play in the assessment and progress of structural damage. Basu and Gupta[1] have shown how the structural damage can thus be estimated in case of mildly non-linear systems by using the response spectrum based techniques for linear systems as in References 2–8.

The pioneering attempts to study the statistics of maxima and the crossings of a particular level by a random function were made by Rice,[9,10] Longuet-Higgins[11] and by Cartwright and Longuet-Higgins.[12] Based on their results, several researchers have studied the response of the structures subjected to random excitations. Udwadia and Trifunac[13] applied the distribution of the largest peak to compute the response spectra. Amini and Trifunac[2,3] presented results on the expected values of the higher order peaks in the

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stochastic response of the structures. However, they ignored the dependence between the ordered peaks, and hence, their predictions had significant errors for the higher order peaks. A further refinement was made by Gupta and Trifunac[14] as they followed the order statistics approach to account for the dependence between the ordered peaks. Their approach did not however consider the statistical dependence between the unordered peaks in any realization of the process. Basu *et al.*[15] proposed more rational statistics of the ordered peak amplitudes on the basis of a Markovian approach. These were shown to contain the results of Gupta and Trifunac[14] as a special case.

Other earlier researchers who have studied the probability distributions of the peaks in a random function in the field of earthquake engineering include Rosenblueth,[16,17] Rosenblueth and Bustamante,[18] Caughey and Gray[19] and Davenport.[20] However, none of them has considered the higher order peak statistics in a random process.

The aim of this paper is to present the distributions of the ordered peaks in a random process by simulating the joint density of the unordered peaks, and to compare these with the distributions given by Gupta and Trifunac[14] and Basu *et al.*[15] For simulation, the peak amplitudes are randomly generated from a joint density function of an assumed functional form. The unknown parameters of this function are then so estimated that the distribution of the generated peak amplitudes matches with the distribution of the maxima given by Cartwright and Longuet-Higgins.[12] A digital experimentation has been carried out to generate the response ensemble of a single-degree-of-freedom (SDOF) oscillator subjected to the artificial accelerogram records, and the statistics of the peaks in this ensemble have been compared with the estimates based on the proposed distributions. A comparative study on the peak factors as estimated from the various formulations and actual response time-histories has also been performed to assess the validity of the assumption of peak independence.

2. DISTRIBUTION OF MAXIMA

Let us consider a random response process, $X(t)$. It is assumed to be stationary, zero mean and Gaussian in nature. $X(t)$ may be the response of a structure to a random excitation, and it may be represented as

$$X(t) = \sum_n C_n \cos(\omega_n t + \phi_n) \tag{1}$$

where, ω_n are the circular frequencies, ϕ_n are the random phases uniformly distributed between 0 and 2π, and C_n are the amplitudes related to the energy spectrum, $S(\omega)$ of $X(t)$ by the following relation

$$\sum_{\omega_n=\omega}^{\omega+d\omega} \tfrac{1}{2} C_n^2 = S(\omega)\,d\omega \tag{2}$$

Extending the work of Rice,[9,10] Cartwright and Longuet-Higgins[12] derived the probability density function for the distribution of maxima of $X(t)$, in terms of the root-mean-square (r.m.s.) value of $X(t)$, a_{rms} and a parameter ε, which is a measure of the width of the energy spectrum, $S(\omega)$. These parameters are defined as follows

$$a_{rms} = m_0^{1/2} \tag{3}$$

and

$$\varepsilon = \left[\frac{m_0 m_4 - m_2^2}{m_0 m_4}\right]^{1/2} \tag{4}$$

where, in general, the nth moment, m_n of the energy spectrum is defined by

$$m_n = \int_0^\infty \omega^n S(\omega)\,d\omega \quad (n = 0, 1, 2, \ldots) \tag{5}$$

The probability density function of the maxima of $X(t)$ as normalized with respect to a_{rms}, is given as

$$p(\eta) = \frac{1}{\sqrt{2\pi}}\left[\varepsilon e^{-\eta^2/2\varepsilon^2} + (1-\varepsilon^2)^{1/2}\eta e^{-\eta^2/2}\int_{-\infty}^{\eta(1-\varepsilon^2)^{1/2}/\varepsilon} e^{-x^2/2}\,dx\right] \tag{6}$$

For $\varepsilon = 0$, this becomes a Rayleigh distribution, and for $\varepsilon = 1$, it becomes a Gaussian distribution. All the peaks of the function, $X(t)$, though marginally distributed in accordance with equation (6), may not be statistically independent. To describe this mutual dependence, it may be convenient to estimate the joint density function between these peaks through digital simulation.

3. JOINT DENSITY FUNCTION AND ORDER STATISTICS

To obtain the simulated joint density function for the peaks in the process, $X(t)$, let us first consider $n+1$ independent random variables, $Y_0, Y_1, \ldots, Y_i, \ldots, Y_n$, each distributed exponentially with parameter β. Thus, each of these random variables has a probability density function given by

$$p_{Y_i}(y_i) = \frac{e^{-y_i/\beta}}{\beta}; \quad i = 0, 1, \ldots, n \tag{7}$$

Further, let the n ordered peaks in $X(t)$ be denoted by $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$. These variables represent ordered peak amplitudes as obtained by arranging the unordered peak amplitudes, $X_1, X_2, \ldots, X_n$ in decreasing order. These unordered amplitudes may be represented as the functions of $Y_0, Y_1, \ldots, Y_n$ by assuming the dependence of the following functional form,

$$X_i = Y_i + Y_0; \quad i = 1, \ldots, n \tag{8}$$

Thus, all the random variables, X_i's are taken to be dependent on each other through the random variable, Y_0.

Since each of the variables, $Y_0, Y_1, \ldots, Y_n$ is independent and identically distributed as in equation (7), their joint density function is given by

$$p(y_0, y_1, \ldots, y_n) = \frac{1}{\beta^{n+1}}\exp\left[-\sum_{j=0}^{n} y_j/\beta\right] \tag{9}$$

Using equation (8) and by transformation of variables (with Jacobian of transformation equal to unity), we can write equation (9) as

$$p(y_0, x_1, \ldots, x_n) = \frac{1}{\beta^{(n+1)}}\exp\left[-\sum_{j=1}^{n} x_j/\beta\right]\exp\left[(n-1)\,y_0/\beta\right] \tag{10}$$

The joint density function of $X_1, \ldots, X_n$ can now be obtained by integrating out the effect of Y_0 in its possible range of variation, say R_y. Thus, we can write

$$p(x_1, \ldots, x_n) = \int_{R_y} \frac{1}{\beta^{(n+1)}}\exp\left[-\sum_{j=1}^{n} x_j/\beta\right]\exp\left[(n-1)\,y_0/\beta\right]dy_0 \tag{11}$$

From equation (11) follows the joint density of $X_{(1)}, \ldots, X_{(n)}$ as

$$p(x_{(1)}, \ldots, x_{(n)}) = \int_{R_y} \frac{n!}{\beta^{(n+1)}}\exp\left[-\sum_{j=1}^{n} x_{(j)}/\beta\right]\exp\left[(n-1)\,y_0/\beta\right]dy_0 \tag{12}$$

To obtain the ith order statistic from equation (12), we must integrate out the effects of the other ordered peaks. It should be noted that, for the orders lower than i, the ordered random variable can vary between the next higher order random variable and infinity, while for the orders greater than i, the range of variation is from the next higher order variable to $x_{(i)}$. It is evident from equation (8) that Y_0 has to be below all the variables, and hence, $X_{(n)}$ which is the highest ordered variable, can vary between y_0 to $x_{(i)}$. Y_0 is however free

to vary over the range (i.e. R_y) from zero to $x_{(i)}$. Following this logic, the density function of the ith ordered random variable can be written as

$$p_{X_{(i)}}(x_{(i)}) = \frac{n!}{\beta^{n+1}} \int_{x_{(2)}}^{\infty} \int_{x_{(3)}}^{\infty} \cdots \int_{x_{(i)}}^{\infty} \int_{x_{(i+2)}}^{x_{(i)}} \int_{x_{(i+3)}}^{x_{(i)}} \cdots \int_{x_{(n)}}^{x_{(i)}} \int_{y_0}^{x_{(i)}} \exp\left[- \sum_{j=1}^{n} x_{(j)}/\beta \right] e^{(n-1)y_0/\beta} \, dx_{(1)} \, dx_{(2)} \cdots dx_{(i-1)} \, dx_{(i+1)} \, dx_{(i+2)} \cdots dx_{(n-1)} \, dx_{(n)} \, dy_0 \tag{13}$$

On successive integration of the above equation, we can write the simplified form of $p(x_{(i)})$ as

$$p(x_{(i)}) = \frac{n!}{(n-i)!(i-1)!} \frac{1}{\beta^2} \int_0^{x_{(i)}} e^{-ix_{(i)}/\beta} (e^{-y_0/\beta} - e^{-x_{(i)}/\beta})^{n-i} e^{(n-1)y_0/\beta} \, dy_0 \tag{14}$$

The expected value of $X_{(i)}$, $E(X_{(i)})$ can be obtained by

$$E(X_{(i)}) = \int_{-\infty}^{\infty} x_{(i)} \, p(x_{(i)}) \, dx_{(i)} \tag{15}$$

It is now possible to obtain the statistics of the ordered peaks by numerically evaluating equations (14) and (15), once the parameter β is estimated.

The above formulation is based on the dependence model as in equation (8) with the variables, Y_i, $i = 0, 1, 2, \ldots, n$ assumed to be exponentially distributed. Other possible variations in this scheme may enhance the mathematical complexity in the formulation and yet not model the joint density function more accurately.

4. ESTIMATION OF β THROUGH DIGITAL SIMULATION

To estimate the parameter, β (as in equations (14) and (15)), a computer simulation has been performed by generating an ensemble of the $(n + 1)$ exponentially distributed random numbers. Using the relation in equation (8), the values for the unordered peaks, $X_1, X_2, \ldots, X_n$ have been obtained. These random numbers in any sample thus have a joint density function as given in equation (11). The size of the ensemble is taken as 50. Univariate distribution for all the $50n$ peaks is now determined. For a given set of n and ε, the parameter β is now varied iteratively to match this univariate density function of the simulated peaks with that corresponding to equation (6).

It has been observed that matching merely by changing the parameter β is not adequate. The curves obtained from the randomly generated peaks also need a 'shift' along the random variable axis. Thus, for

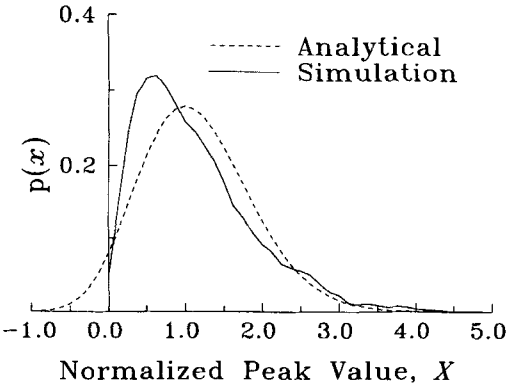

Figure 1. Comparison of analytical and simulated p.d.f. of peaks for $\varepsilon = 0{\cdot}4$, $n = 100$ and $\beta = 0{\cdot}56$

Table I. Estimated parameters of joint density function from simulation

ε	β	s
0·0	0·54	0·0
0·2	0·54	0·0
0·4	0·56	0·0
0·6	0·61	0·0
0·8	0·78	0·5
1·0	0·88	1·32

a given set of n and ε, iteration is done over both the parameters, β and shift, s. On taking n between 20 and 160, and ε between 0·0 to 1·0 at an interval of 20 and 0·2, respectively, it has been seen that β and s remain invariant with n. Figure 1 shows the matching of the two univariate density curves, one obtained from the simulated peaks and the other from the theoretical distribution, for a typical case with $n = 100$ and $\varepsilon = 0{\cdot}4$. For this case, the values of β and s are respectively obtained as 0·56 and 0·0. The results of β and s for the other values of ε are presented in Table I. Since most seismic responses are narrow banded (with $\varepsilon \leqslant 0{\cdot}4$), $\beta = 0{\cdot}55$ with 'no shift' should give reasonably good approximations of the joint density in the response peaks.

5. RESULTS AND DISCUSSION

The random process representing the response of a structure to earthquake excitations has been generated by obtaining SDOF oscillator response to synthetic accelerograms. The accelerograms are for the Dumbarton Bridge site (near Coyote Hills) and corresponding to the 1989 Loma Prieta Earthquake (see Reference 21 for details). The oscillator is assumed to have a period of 0·5 s and a damping ratio of 2 per cent. 140 response time histories have been considered with the total duration, T of 40·96 s. These response time histories were non-stationary in nature, due to which a_{rms} varied with time. Hence, the peak statistics have been obtained based on an intermediate stationary zone which represents a constant growth of $X^2(t)$ with time. Figure 2 shows this growth for seven typical time histories with total number of peaks varying between 80 and 100. It is seen that the curves have almost constant gradient between $t = 8$ s and $t = 32$ s. Hence, all the time histories are truncated in the beginning ($t < 8$ s) and at the end ($t > 32$ s) to obtain an ensemble of a relatively more stationary process. The values of ε for the truncated time histories have been computed from the ratio of the number of negative peaks to the total number of peaks (see Reference 12). For finding the order statistics of the peaks, 64 records with average ε value of 0·4 and total number of peaks, $n = 54$ have been considered. Figures 3, 4 and 5, respectively, show the comparison of the histograms for the first, fifth and ninth order peaks respectively with the probability densities of these ordered peaks as estimated from the simulated joint density function. Since the non-stationarity trends are not completely eliminated by the truncation of the beginning and the end portions, the histograms have been plotted after scaling the experimentally obtained peak amplitudes such that the mean of the histogram for the first order peak matches with the mean of the simulated distribution of the first order peak. This takes care of the approximation in the r.m.s. value due to non-stationarity and sensitivity of a_{rms} on the length of the time window considered (see Reference 15). The histograms are found to match with the simulated density functions reasonably well. Table II shows the comparison of expected values and standard deviations of the ordered peaks ($i = 1, 5, 9$) as obtained from the histograms with those based on the different theoretical formulations. It is seen that the expected values from the digital simulation and the Markov theory[15] match with the histogram results quite well, while the results based on the independence of peaks are on the higher side. For the largest peak ($i = 1$) , however, Markov estimates are slightly lower while those based on the peak independence match rather well. Moreover, for $i = 9$, the mean corresponding to the histogram is a little lower compared to those from the theoretical formulations. This suggests that the same r.m.s. value as

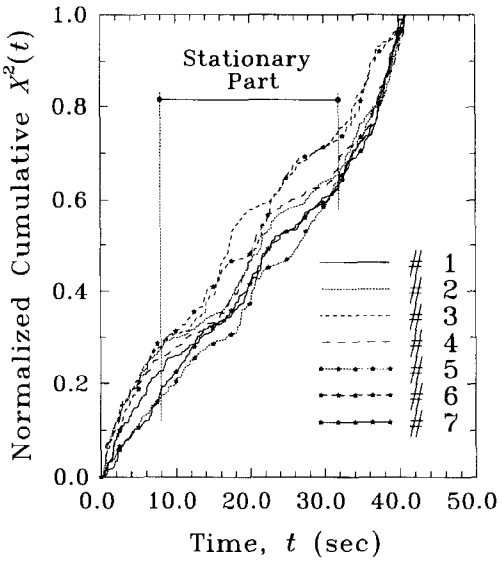

Figure 2. Growth of $X^2(t)$ with time t for the SDOF system response

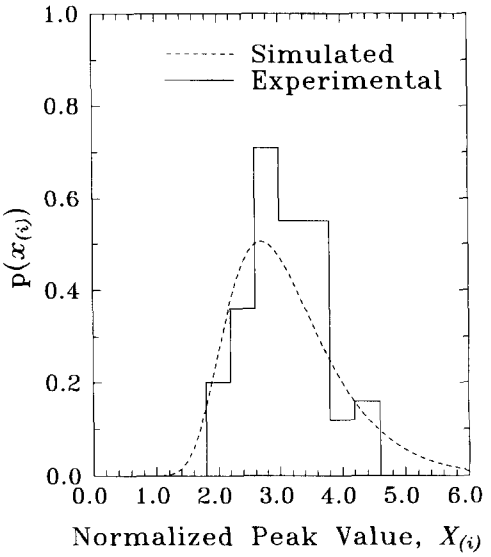

Figure 3. Comparison of simulated and experimental p.d.f. for first order peak with $\varepsilon = 0{\cdot}4$ and $n = 54$

used to normalize the first order peak data in plotting the histograms was not good for normalizing the ninth order peak data as well. This observation is however not true for the fifth order peak, and thus, it might imply that the r.m.s. value true for the first order peak is applicable for some, but not all, higher order peak amplitudes in case of the non-stationary processes. The number of these higher order peaks might depend on the total number of peaks in the process because with $n = 80$, the agreement between the ninth order peaks was found to be quite good by Basu *et al.*[15] in case of the Markov theory. Further, the standard deviations for the simulated joint density results are seen to tally reasonably well with those from the histogram data. The standard deviations for the Markov theory and independence assumption are in good agreement with each

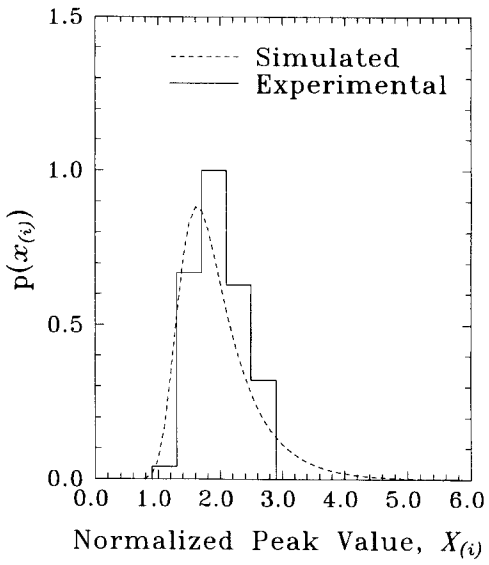

Figure 4. Comparison of simulated and experimental p.d.f. for fifth order peak with $\varepsilon = 0{\cdot}4$ and $n = 54$

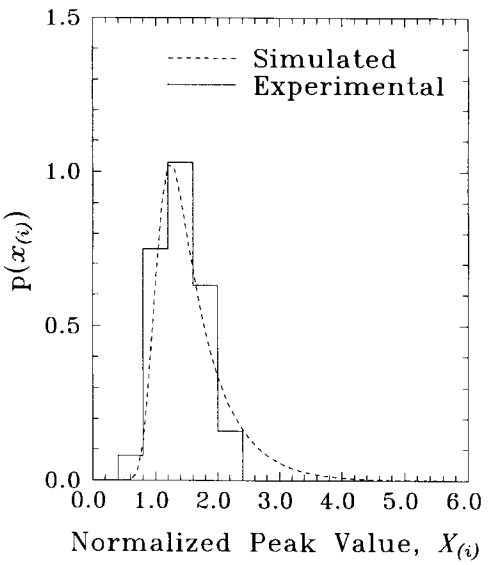

Figure 5. Comparison of simulated and experimental p.d.f. for ninth order peak with $\varepsilon = 0{\cdot}4$ and $n = 54$

Table II. Comparison of mean and S.D. from histogram and different theories

	Simulation		Histogram		Markov		Independence	
i	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
1	3·09	0·89	3·04	1·11	2·78	0·43	2·97	0·41
5	1·96	0·61	1·96	0·70	1·96	0·21	2·18	0·20
9	1·60	0·58	1·37	0·48	1·63	0·17	1·87	0·17

other, but are low compared to the histogram results. This means that both of these models predict smaller dispersion in the ordered peak amplitudes than what has been observed in this digital experimentation.

A parametric study has been carried out to compare various analytical formulations available for estimating the amplitudes of ordered peaks in random processes. The formulation based on the simulated joint density presented in this paper has been considered along with those based on the Markov theory (see Reference 15) and on the statistical independence of peaks (see References 22 and 14). First, the variation of the expected value of the largest peak with the total number of peaks, n has been considered for $\varepsilon = 0{\cdot}4$. The three curves for these formulations (see Figure 6) show a similar increasing trend with the increase in n. It may be noted that the curve for the assumption that the peaks are independent is almost parallel to that for the Markov theory. Further, it is obvious from Figure 6 that the simplified approach of taking the same peak factors for all the modes of vibration in multi-degree-of-freedom structures (see References 23 and 24, for example) may be reasonable only for stiff structures subjected to the long duration ground motions. To illustrate, if the number of peaks in the fundamental mode of vibration is around 30 compared to 120 in a higher mode of vibration, the peak factor based on $n = 120$ may give about 15% error (as per the Markovian estimates[15]) in scaling the spectrum amplitudes in the fundamental mode.

Figure 7 shows the comparison of the probability density curves for the largest peak, $n = 100$ and $\varepsilon = 0{\cdot}4$. Here, again, the density curves from the Markov theory and assumption that peaks are independent are found to have excellent match. The curve obtained from the simulated joint density function is 'flatter' compared to the other two.

The variation in the expected values with ε has been shown in Figure 8 for the largest peak and $n = 100$. All the three curves show that the expected peak amplitudes remain almost invariant with ε for the practical range of ε for a structural response. Thus, for estimation of the expected peak response, narrow band approximation of the response process appears to be excellent.

To see how the expected values of the different ordered peaks vary for the three formulations, curves have been plotted in Figure 9 for $n = 100$ and $\varepsilon = 0{\cdot}4$. It is seen that for the first few peaks, the results from the independence assumption are close to those from the simulated joint density while for the higher order peaks ($i > 5$), there is excellent agreement between the Markov estimates and the results from the simulated density. Hence, for the estimation of the largest peak, the assumption of independence may be quite reasonable while for the higher order peaks, this assumption leads to conservative estimates. Further, the widening gap between the curves from independence assumption and from Markov theory/simulation density, with the

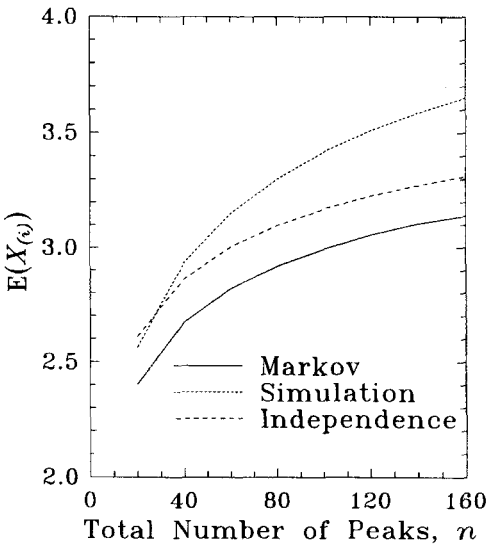

Figure 6. Variation in $E(X_{(i)})$ with n for $i = 1$ and $\varepsilon = 0{\cdot}4$

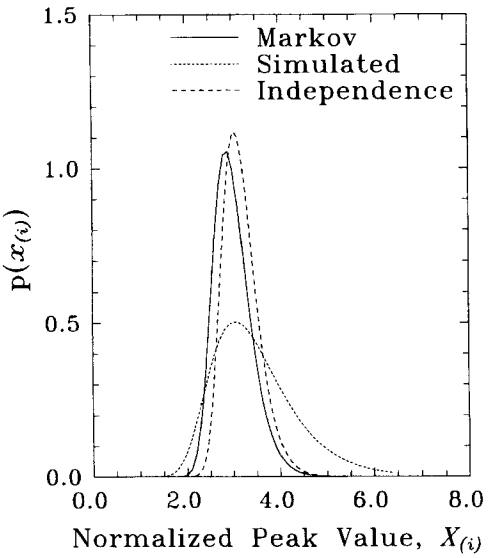

Figure 7. Comparison of density functions for $i = 1$, $\varepsilon = 0{\cdot}4$, and $n = 100$

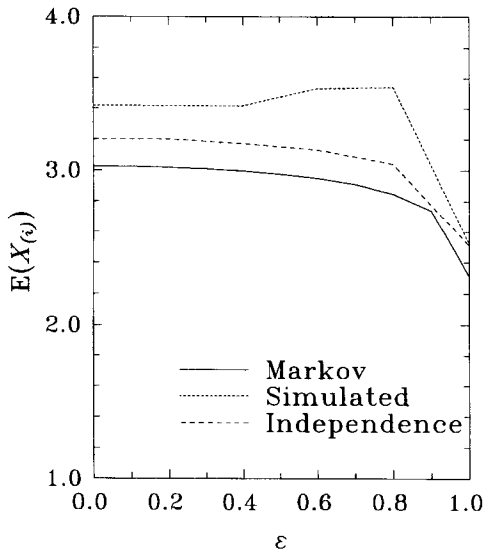

Figure 8. Variation in $E(X_{(i)})$ with ε for $i = 1$ and $n = 100$

increase in the order of peaks, raises the doubts over the validity of independence assumption in case of the higher order peaks. On the other hand, there is a tendency of the Markov estimates to be slightly lower for the first few peaks.

A more critical examination of the assumption of independence requires the comparison of the estimated peak factors from various formulations with the 'actual' peak factors for a few more earthquake processes. To this end, two additional motions with significantly different durations from that at the Dumbarton bridge have been considered. These are (i) synthetically generated motion at Mexico City site for the 1985 Michoacan earthquake with 80·96 s duration (see References 6 and 7 for details), and (ii) recorded S60E component at Santa Fe Springs site for the first aftershock of 1987 Whittier Narrows earthquake with 14·02 s duration. Whereas the synthetic motion for the Michoacan earthquake is longer in duration, the recorded

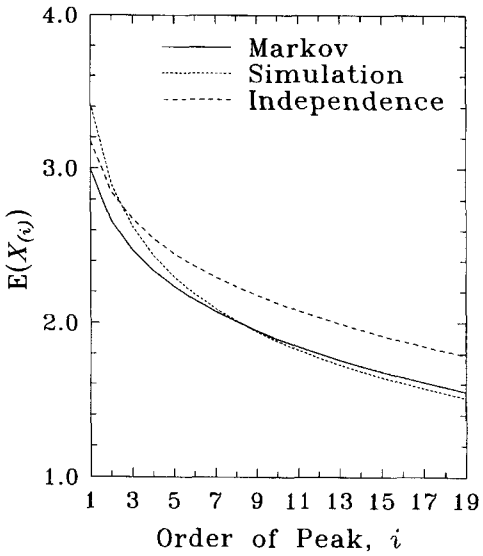

Figure 9. Variation in $E(X_{(i)})$ with i for $\varepsilon = 0{\cdot}4$ and $n = 100$

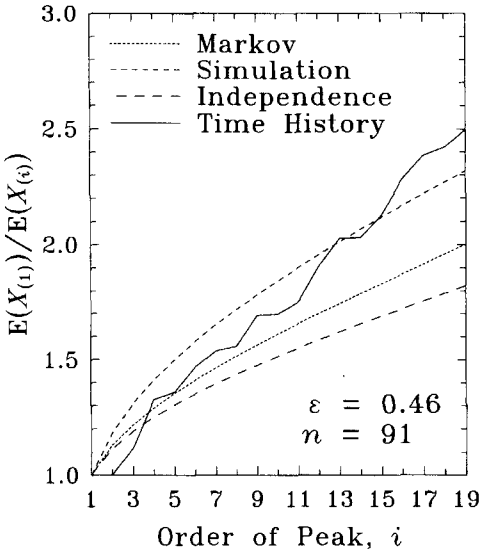

Figure 10. Variation in $E(X_{(1)})/E(X_{(i)})$ with i for 1% damping oscillator response to Michoacan earthquake

aftershock motion is of short and impulsive nature. For convenience in computing the 'actual' peak factors, we assume that these records represent the 'expected' ground motions at the respective sites. Response time histories for these ground motions have been generated by considering two SDOF oscillators having the same natural period of 0·5 s, and damping values of 1% and 5% of the critical respectively. Since the 'actual' peak factors are sensitive to the choice of a_{rms}, the comparison of peak factors has been made in a normalized sense by considering the variation of $E(X_{(1)})/E(X_{(i)})$ with the order of peak, i. This ratio may be interpreted as the reduction factor to be applied on the conventional response spectrum amplitudes (for the largest peak) to obtain the amplitudes for the ith order of peak.

Figures 10 and 11 show the results for the oscillators with 1% and 5% damping ratios respectively in the case of Michoacan earthquake. The number of peaks, n in the two response processes is found to be 91 and

71, respectively. Similarly, the values of ε are calculated to be 0·46 and 0·62, respectively. It is seen in the figures that the curves based on the assumption of independence are consistently lower than the other curves except for the first few orders of peaks. The overall matching of the curves based on the time histories is better with those based on the Markov formulation[15] and simulated joint density. Similar observations are also made in Figures 12 and 13 for the Whittier Narrows aftershock case. In this case, the values of n are 29 and 33, and the values of ε are 0·75 and 0·85, respectively, for the oscillators of 1 per cent and 5 per cent damping ratios. Results have been presented in Figures 12 and 13 for fewer orders of peaks because a lesser number of peaks may be relevant for design in case of processes with small n.

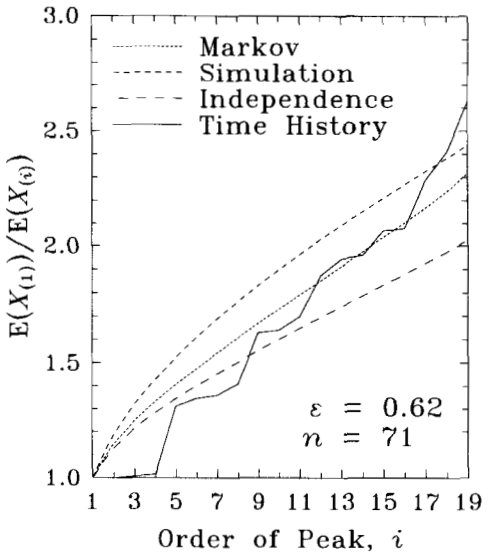

Figure 11. Variation in $E(X_{(1)})/E(X_{(i)})$ with i for 5% damping oscillator response to Michoacan earthquake

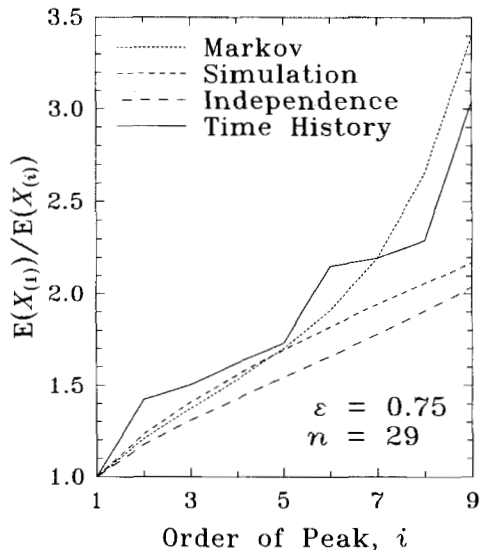

Figure 12. Variation in $E(X_{(1)})/E(X_{(i)})$ with i for 1% damping oscillator response to Whittier Narrows aftershock

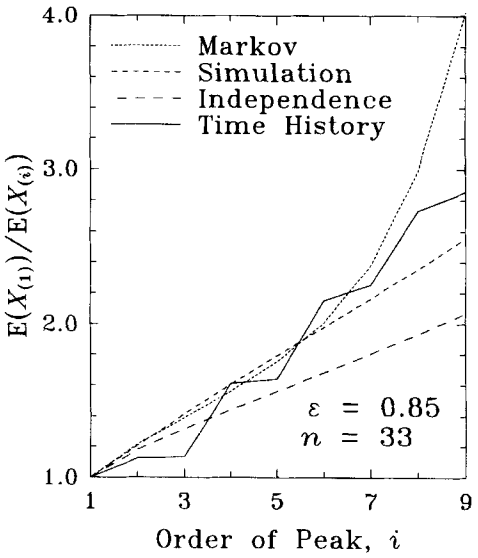

Figure 13. Variation in $E(X_{(1)})/E(X_{(i)})$ with i for 5% damping oscillator response to Whittier Narrows aftershock

6. CONCLUSIONS

A theory on the order statistics of the peaks in a random process has been formulated by taking into account the statistical dependence between these peaks. The estimates from this theory are seen to match the statistics obtained from a synthetically generated ensemble of a response process reasonably well. Further, the expected values of the ordered peaks from this theory are in close agreement with the results from the Markov theory, especially for the higher order peaks.

It has been seen by considering responses of SDOF oscillators to earthquakes of widely different durations that the assumption of independence among peaks may not be valid for all orders of peaks. This assumption is found to be reasonably good for the largest few peaks while this may lead to conservative estimates for the higher order peak amplitudes. Further, the narrow-band approximation of the usually encountered structural response may be excellent for estimating the ordered peak amplitudes.

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